統計

1.(20%) Let $X_1, \cdots, X_n$ be a sample from the probability density function

$$f(x|\mu,\lambda) = (\frac{\lambda}{2\pi x^3})^{1/2} \exp\{-\lambda(x-\mu)^2/2\mu^2 x\}, x > 0.$$

Find the maximum likelihood estimators of μ and λ.

2.(20%) Let $X_1, \cdots, X_n$ be i.i.d. $N(\theta, \sigma^2)$, and let θ have a double exponential distribution, that is, $\pi(\theta) = e^{-|\theta|/a}/2a$, a known. Find the mean of the posterior distribution of θ.

3.(20%) Let $X_1, \cdots, X_n$ be a sample from a $N(\mu_x, \sigma_x^2)$, and let $Y_1, \cdots, Y_m$ be a sample from a $N(\mu_y, \sigma_y^2)$. Find the likelihood ratio test for

$$H_0 : \mu_x = \mu_y \ \text{ versus } \ H_1 : \mu_x \neq \mu_y,$$

with the assumption that $\sigma_x^2 = \sigma_y^2$.

4.(20%) Let $X_1, \cdots, X_n$ be a random sample from a $N(\theta, \sigma^2)$ population. Find an unbiased test for

$$H_0 : \theta_1 \leq \theta \leq \theta_2 \ \text{ versus } \ H_1 : \theta < \theta_1 \text{ or } \theta > \theta_2.$$

5.(20%) Find the regression line $y = \alpha + \beta x$ such that the sum of horizontal distances between the line and $(X_1, Y_1), \cdots, (X_n, Y_n)$ is minimized.