資格考 (Algebra 2004-Fall)

1. Prove or disprove that there exists an infinite field F of characteristic 92011. (10 %)
2. Let field F be an algebraic extension of a field K of characteristic 0 such that every polynomial in $K[x]$ has a root in F. Prove or disprove that F is an algebraic closure of K. (15 %)
3. Prove or disprove that every element in a finite field Z/pZ may be written as the sum of two squares, where Z denotes the ring of all rational integers and p a prime in Z. (15 %)
4. Let G be an abelian group and I, J be any infinite subgroups of G with G/I and G/J being finite groups. Prove or disprove that $I \cap J$ is an infinite subgroup of G. (15 %)
5. Let D be a Dedekind domain and α, β elements in D. Prove or disprove that a greatest common divisor of $\{\alpha, \beta\}$ always exists. (15 %)
6. Prove or disprove that the set consisting of zero and all zero divisors in a commutative ring R with identity $1 \neq 0$ contains at least one prime ideal. (15 %)

7. Let E be a principal ideal domain and α, β nonzero nonunit elements in E. Suppose $\pi_{\alpha\beta}(E') \supseteq (E/\alpha\beta E)^*$. Prove or disprove that $\pi_{\alpha}(E') \supseteq (E/\alpha E)^*$, where E' is a subset of E, $\pi_x : E' \rightarrow E/xE$ the canonical map and $(E/xE)^*$ the unit group of E/xE. (15 %)