6. Let matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

(1) Compute A^2, A^3, and A^n.

(2) If $f(t) \in \mathbb{R}[t]$ is such that $f(A)$ is equal to zero matrix, then show that $(t^3 - 4t)|f(t)$.

(10 pts)

7. Let G be a finite group of order 45. Then

(1) Find the numbers of Sylow 3-subgroup, Sylow 5-subgroup of G.

(2) Show that G is a direct product of two normal subgroups of G.

(3) Show that G is an abelian group.

(15 pts)

8. Let G be a group of order 10. Then show that

(1) There exists two elements $a, b \in G$ such that $o(a) = 2, o(b) = 5$.

(2) (b) is a normal subgroup of G.

(3) If $aba^{-1} = b^n$ where $0 \leq n \leq 4$, then compute $n =$?

(15 pts)