

factor of β , then $p^2(X) \nmid \beta$. Show that

$$\mathcal{O} = A[\beta^{\frac{1}{2}}] = \{f + g\beta^{\frac{1}{2}} \mid f, g \in A\}.$$

- (d) Assume that $\beta = X^2 + 1$. Is \mathcal{O} a P.I.D.? Prove or disprove it.
- (6) (20%) Let K be a field and let $K[X]$ be the polynomial ring. Denote $R_i = K[X]/(X^i)$, $i = 1, 2, \dots$. In particular, $R_1 = K[X]/(X) \simeq K$ and $R_2 = K[X]/(X^2)$.
- (a) Show that R_1 , considered as an R_1 -module, is an injective module.
- (b) Show that R_2 , considered as an R_2 -module, is an injective module.