5. Let p be a prime, K a finite field of characteristic p, a an element of K, and $P(X) = X^p - X + a$. Let C be an algebraic closure of K.

    a. (5 pts.) Let α be a root of $P(X)$ in C. Determine the other roots of $P(X)$ in C.

    b. (5 pts.) Deduce that either the polynomial $P(X)$ has all its roots in K, or it is irreducible in $K[X]$.

    c. (5 pts.) Let $a \neq 0$ in $\mathbb{F}_p$. Show that the splitting field of the polynomial $P(X)$ over $\mathbb{F}_p$ is an extension of degree p of $\mathbb{F}_p$.

    d. (5 pts.) Let n be an integer. Show that the polynomial $X^p - X + n$ is irreducible in $\mathbb{Q}[X]$ for an infinite number of values of n.