counter example.

(c) Let K be the field of fraction of A and let $L = K(y)$ be a finite separable degree $d > 1$ extension of K. Moreover y satisfies the equation

$$x_1 Y^d + g(x_1, \ldots, x_n, Y) = 0$$

where $g(x_1, \ldots, x_n, Y) \in k[x_1, \ldots, x_n, Y]$ with Y-degrees less than d. Let $B = k[x_1, \ldots, x_n, y]$.

(i) (5 %) Is it true that B is a finitely generated A-module?
(ii)(9 %) Let C be the integral closure of A in L. Show that there exists a basis $v_1, \ldots, v_d$ of L over K such that $C \subseteq D = \sum_{j=1}^{d} A\, v_j$ and that D/C is a torsion A-module.

(d) (7 %) Let $\mathfrak{q}$ be a maximal ideal of C and let $\mathfrak{p} = \mathfrak{q} \cap A$. Show that $\mathfrak{p}$ is a maximal ideal of A and $C/\mathfrak{q}$ is a finite extension of $A/\mathfrak{p}$.

(e) (8 %) Let $\mathfrak{q}$ be a prime ideal of C such that $\mathfrak{q} \cap A = (f)$ for some irreducible polynomial f. Show that $C_{\mathfrak{q}}$ is the integral closure of $A_{(f)}$ in L and $C_{\mathfrak{q}}$ is also a discrete valuation ring.

(4) Let L be a finite Galois extension of the field K with Galois group $G = \mathrm{Gal}(L/K)$. Let $n \geq 1$ be positive integer.

(a) (9 %) Prove that $L \otimes_K L \simeq \prod_{\sigma \in G} \sigma L$ as L-algebras.

(b) Let $\mathrm{GL}_n(L)$ be the group of $n \times n$ invertible matrices with entries in L.
(i) (3 %) Verify that G acts on $\mathrm{GL}_n(L)$.
(ii) (12 %) A cross homomorphism of G into $\mathrm{GL}_n(L)$ is a map $\phi : G \to \mathrm{GL}_n(L)$ satisfying the relation $\phi(\sigma\tau) = \phi(\sigma)\sigma\phi(\tau)\forall\sigma,\tau \in G$. Show that for any cross homomorphism $\phi : G \to \mathrm{GL}_n(L)$, there exists an $a \in \mathrm{GL}_n(L)$ which depends on ϕ such that $\phi(\sigma) = a^{-1}\sigma a$.