分析 2005. 2.

歸檔

20% 1. Let $f \in C(I)$, $g \in BV(I)$, $I=[a,b]$ and let $F(x)=\int_a^x f\,dg$, $x \in [a,b]$. Show that if g is continuous at x_0 then F is also continuous at x_0.

20% 2. Let $E_n \subset [0,1]$ be Lebesgue measurable sets and $m(E_n) \to 1$, where m is the Lebesgue measure. Show that there is a subsequence $\{E_{n_j}\}_{j=1}^{\infty}$ so that $m(\bigcap_{j\geq 1} E_{n_j}) > 0$.

20%3. Let $f_n : E \to \mathbf{R}$ so that $f_n \to f$ pointwise on E, where $E \subset X$, X is a space and E is uncountable. Prove that there is an infinite set $A \subset E$ so that $f_n \to f$ uniformly on A.

20%4. Given a measure space (X, M, μ), $0 < \mu(X) < \infty$, $f \in L^{\infty}(\mu)$ so that $\|f\|_{\infty} > 0$. Let $\alpha_n = \int_X |f|^n \, d\mu$, $n = 1,2,3,\ldots$.Show $\frac{\alpha_{n+1}}{\alpha_n} \to \|f\|_{\infty}$.

20% 5. If $\{f_n\} \subset L^p(\mathbf{R})$, $1 < p \leq \infty$, and $\|f_n\|_p \leq K < \infty$ for $n = 1,2,\ldots$. Show that there is a subsequence $\{f_{n_j}\}_{j=1}^{\infty}$ of $\{f_n\}_{n=1}^{\infty}$ and a function $f \in L^p$ so that $f_{n_j} \to f$ weakly, i.e., $\int_{\mathbf{R}} f_{n_j} \varphi \, dm \to \int_{\mathbf{R}} f \varphi \, dm$ $\forall \varphi \in L^q$, where $\frac{1}{p} + \frac{1}{q} = 1$.