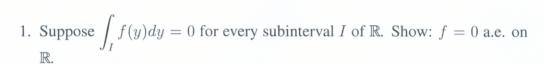


DEPARTMENT OF MATHEMATICS NATIONAL CENTRAL UNIVERSITY

Ph. D. Qualifying Examination

Fall, 2002

Analysis



2. If $f \in L^1(-\infty, \infty)$ then show:

$$\lim_{n \to \infty} \int_{-\infty}^{\infty} f(x) \cos nx \, dx = 0$$

3. Let F(x,y) be continuous and bounded on the unit square 0 < x < 1, 0 < y < 1. Show: $f(y) = \overline{\lim}_{x \to 0+} F(x,y)$ is measurable on (0,1).

4. Suppose f, f_n are of bounded variation on [a, b], $n = 1, 2, \cdots$ and

$$V(f_n - f, a, b) \to 0$$

(where V(f, a, b) = the total variation of f on [a, b]) Prove: There is a subsequence $n_k \to \infty$ such that

$$f'_{n_k} \longrightarrow f'$$
 a.e.

5. Let $f \in L(\mathbb{R}^n)$. Show

$$\lim_{|h|\to 0} \int_{\mathbb{R}^n} |f(x+h) - f(x)| dx = 0$$

6. Let $f, g \in L^1(\mathbb{R}^1)$. Show:

(a) f(x-t)g(t) is a measurable function on \mathbb{R}^2 .

(b)
$$\phi(x) = \int_{-\infty}^{\infty} f(x-t)g(t)dt$$
 is in $L^1(\mathbb{R}^1)$