6.(a) Let $K \in L^1(R^n)$, $\int_{R^n} K = 1$ and $K_\epsilon(x) = \epsilon^{-n} K(\frac{x}{\epsilon})$. Define

$$f_\epsilon(x) = (f * K_\epsilon)(x) = \int_{R^n} f(t) K_\epsilon(x-t)\, dt, x \in R^n.$$

Prove that if $f \in L^p(R^n), 1 \le p < \infty$, then $||f_\epsilon - f||_p \to 0$ as $\epsilon \to 0$.

(b) Prove that C_0^∞ is dense in $L^p(R^n)$ for $1 \le p < \infty$.

√ 7.(a) Let f be a nonnegative function which is integrable over a measurable E. Prove that given any $\epsilon > 0$ there is $\delta > 0$ such that for every set $A \subset E$ with $m(A) < \delta$ we have

$$\int_A f < \epsilon.$$

(b) Let f be a Lebesgue integrable function on $[a, b]$, and

$$F(x) = F(a) + \int_a^x f(t)dt.$$

Prove that F is absolute continuous on $[a, b]$ and $F'(x) = f(x)$ for almost all x in $[a, b]$.