6. Prove that if a subset A of $[0, 1]$ has measure zero then

$$A^2 = \{x^2 : x \in A\}$$

has measure zero, too.

7. Suppose that f is a real-valued differentiable function on $[0, 1]$. Prove that its derivative function f' is Lebesque measurable on $[0, 1]$.

8. Let μ_n be a non-decreasing sequence of measures defined on a measurable space $(X, \mathcal{A})$ in the sense that $\mu_n(A) \uparrow \mu(A)$ for all A in $\mathcal{A}$.

   (a) Prove that μ is a measure on $(X, \mathcal{A})$ with respect to which all μ_n are absolutely continuous.

   (b) On any fixed set A of finite μ measure, let f_n denote the Radon–Nikodym derivative of μ_n with respect to μ. Prove that almost everywhere (with respect to μ, and thus all μ_n) on A, $f_n \uparrow 1$.

9. Suppose that f is a real-valued continuous function on $[0, 1]$. Show that $||f||_{L^p} \longrightarrow ||f||_{L^\infty}$ as $p \longrightarrow \infty$.