

國立中央大學數學系
博士班資格考試
〈圖論〉試題
Fall 2003

NOTE: 請寫下解題的詳細過程。

useful definition

A *snark* is a cyclically 4-edge connected cubic graph of girth at least 5 that has chromatic index 4.

Problem 1 Please describe Dijkstra's algorithm for determining the shortest path between two specified vertices of a simple connected weighted graphs. (10 分)

Problem 2 Show that a graph G is chordal if and only if every minimal vertex cut of G is a clique. (10 分)

Problem 3 Show that a plane triangulation has a 3-face coloring if and only if it is not K_4 . (10 分)

Problem 4 (20 分)

(a) Prove or disprove that if G is a simple graph with number of vertices $n(G) \geq 3$, and G has at least $\alpha(G)$ vertices of degree $n(G) - 1$, then G is Hamiltonian.

(b) Prove that if $\kappa(G) \geq \alpha(G)$, then G has a Hamiltonian cycle (unless $G = K_2$).

Problem 5 Show that there is no 6-connected planar graph. (10 分)

Problem 6 Show that for a simple graph G of order n and size m , the chromatic polynomial $f(G; \lambda)$ is a monic polynomial of degree n in λ with integer coefficients and constant term zero, and its coefficients alternate in sign and the coefficient of λ^{n-1} is $-m$. (20 分)

Problem 7 Show that the Petersen graph P is the smallest snark and it is the unique snark on 10 vertices. (10 分)

Problem 8 Show that the Ramsey numbers $R(3, 3) = 6$ and $R(3, 4) = 9$. (10 分)

Good Luck !