

圖論

- ① (15%) ① If G is k -critical (i.e., $\chi(G) = k$, $\chi(G - x) = k - 1$ for every $x \in V(G)$, where $\chi(G)$ is the chromatic number of a graph G), show that the minimum degree $\delta(G) \geq k - 1$.
- ② If $\chi(G) = k$, show that G contains at least k vertices each of which has degree at least $k - 1$.
- ② (10%) A graph G is 3-edge connected. Show that every edge of G is the intersection of two cycles in G .
- ③ (10%) Show that if $\text{diam } G \geq 3$ ($\text{diam } G$ is the diameter of G) then $\text{diam } G^c \leq 3$.
- ④ (20%) ① Show that every tournament has a directed Hamiltonian path.
- ② Show that every tournament contains a vertex from which every vertex is reachable by a directed path of length at most 2.