§本試題使用 **D. B. West** 書中的符號與定義。

1. Show that if G is bipartite, then $\chi'(G) = \Delta(G)$. (10%)

2. **(a)** State and prove the Brooks' Theorem. (10%)
   (b) Show that Brooks' Theorem is equivalent to the following statement: every
      (k-1)-regular k-critical graph is a complete graph or an odd cycle. (10%)

3. Prove that if G is a 3-rugular graph then $\kappa(G) = \kappa'(G)$. (10%)

4. Show that the vertices of a digraph D can be covered using at most $\alpha(G)$ pairwise
   vertex-disjoint dipaths. (10%)

5. Show that if $\binom{n}{p} 2^{1-\binom{p}{2}} < 1$, then $R(p, p) > n$. (10%)

6. **(a)** Show that $R(3,4) = 9$. (5%)
   (b) Show that $R(3,5) = 14$. (5%)

7. **(a)** Show that $\alpha(G) \geq \sum_{v \in V(G)} \frac{1}{d(v)+1}$ for every graph G. (5%)

   (b) Using (a) to show that $\alpha(G) \geq \frac{n(G)}{\bar{d}+1}$, where $\bar{d} = \frac{2e(G)}{n(G)}$. (5%)

8. Show that for any positive integer k, there exists a triangle-free graph with chromatic
   number k. (10%)

9. **(a)** For which k there exists a planar k-regular graph? Show your work. (5%)
   (b) For which k there are no planar k-regular graphs? Show your work. (5%)