國立中央大學數學系博士班資格考<圖論>試題　　　Fall 2006

注意:每個問題必需證明或說明清楚,並盡可能證明所有引用的定理。

1. Suppose G is a bipartite graph. Give a necessary and sufficient condition for G having a perfect matching and prove it. 　　　(20%)

2. Prove that every tournament has a vertex from which every vertex is reachable by a directed path of length at most 2. 　　　(15%)

3. A graph $G=(V,E)$ with n vertices is Hamiltonian-connected if every two vertices of G are connected by a Hamiltonian path. Prove that if $|E|\geq(n-1)(n-2)/2+3$ then G is Hamiltonian-connected. 　　　(20%)

4. Suppose G is a planar graph with the girth at least 4. Prove G has a proper 4-coloring. 　　　(15%)

5. True or False. (If the statement is true, prove it; if it is false, give a counterexample) 　　　(10%×3)
   (a) If T is a tree with the diameter at least 3 then the complement of T is connected.
   (b) If a graph G is bipartite then the chromatic index of G is equal to the maximum degree of G.
   (c) If the chromatic number of a graph G is 4 then G contains a subgraph which is isomorphic to the complete graph of order 4.